Finite element modelling of radio-frequency heating in fusion devices

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Summary

In this work, we present an iterative method to account for non-local effects, such as spatial dispersion, in complex geometries when modelling radio frequency heating in magnetic confinement fusion devices. The proposed method combines the finite element method with spectral methods and has been implemented in the existing code FEMIC and validated against existing heating codes with good agreement.

1. Introduction

In magnetic confinement fusion devices, the fuel needs to be heated to hundreds of millions degrees. At these temperatures, the hydrogen gas is a fully ionized plasma, and the ions and electrons are confined by a strong external magnetic field. To reach fusion relevant temperatures, radio-frequency heating tuned to a cyclotron frequency is often used. To predict how the waves propagate and are dissipated, the plasma can be modelled as a static dielectric characterized by the dielectric response tensor $\widetilde{\boldsymbol{K}}$, which in general is a non-isotropic integro-differential operator [1]. When simulating such non-local couplings, the electromagnetic wave equation is often discretized using spectral methods [2, 3]. Doing so, it is possible to model physical phenomena, such as spatial dispersion and mode-conversion to electrostatic waves. These phenomena are normally difficult to model with local discretization techniques, such as the finite element method. However, spectral methods are not well suited to describe the complex geometries of wall and antenna components, and are therefore unable to describe the interaction between the high-power waves and plasma facing components [4]. In this work, we present an iterative method that adds non-local contributions to a finite element model with an otherwise local dielectric response tensor. This method retains the efficiency and versatility of the finite element method, while retaining important physical effects.

2. Theory

In general, the electromagnetic wave equation in a non-local medium in frequency domain can be written as

$$\nabla \times (\nabla \times \boldsymbol{E}) - \frac{\omega^2}{c^2}\widetilde{\boldsymbol{K}}[\boldsymbol{E}] = \mathrm{i}\omega\mu_0 \boldsymbol{J}_{\mathrm{ext}}, \tag{1}$$

where ω is the wave frequency, c is the speed of light in vacuum, and $\boldsymbol{J}_{\mathrm{ext}}$ is the source current. In the context of this work, $\boldsymbol{J}_{\mathrm{ext}}$ represents the antenna current or image currents on passive wall components. The dielectric response $\widetilde{\boldsymbol{K}}$ is split into two parts, namely $\widetilde{\boldsymbol{K}} = \boldsymbol{K} + \delta\widetilde{\boldsymbol{K}}$, where $\boldsymbol{K}$ is a local, algebraic approximation of the non-local operator $\widetilde{\boldsymbol{K}}$, and $\delta\widetilde{\boldsymbol{K}}$ include the dispersive effects missing from the algebraic approximation $\boldsymbol{K}$. The operator $\delta\widetilde{\boldsymbol{K}}[\boldsymbol{E}]$ can be expressed as a current density

$$\delta\boldsymbol{J}_{\mathrm{ind}} = -\mathrm{i}\omega\varepsilon_0\delta\widetilde{\boldsymbol{K}}[\boldsymbol{E}], \tag{2}$$

so that the wave equation can be written as

$$\nabla \times (\nabla \times \boldsymbol{E}) - \frac{\omega^2}{c^2}\boldsymbol{K}\cdot\boldsymbol{E} = \mathrm{i}\omega\mu_0(\boldsymbol{J}_{\mathrm{ext}} + \delta\boldsymbol{J}_{\mathrm{ind}}). \tag{3}$$

The current density $\delta\boldsymbol{J}_{\mathrm{ind}}$ is not easily discretized using the finite element method, and is therefore evaluated using spectral methods, such as continuous wavelet transforms [5] or Fourier decomposition [6]. Due to the different discretization techniques, equation (3) is not solved self-consistently, but iteratively. The electric wave field in the k:th iteration step is obtained from

$$\boldsymbol{E}_{\mathrm{A}}^{(k)} = \mathrm{A}\left[\boldsymbol{E}^{(k)}, \boldsymbol{E}^{(k-1)}, \dots, \boldsymbol{E}^{(1)}\right], \tag{4}$$

$$\delta\boldsymbol{J}_{\mathrm{ind}}^{(k)}\left(\boldsymbol{E}_{\mathrm{A}}^{(k)}\right) = -\mathrm{i}\omega\varepsilon_0\delta\widetilde{\boldsymbol{K}}\left[\boldsymbol{E}_{\mathrm{A}}^{(k)}\right], \tag{5}$$

$$\nabla \times \left(\nabla \times \boldsymbol{E}^{(k+1)}\right) - \frac{\omega^2}{c^2}\boldsymbol{K}\cdot\boldsymbol{E}^{(k+1)} = \mathrm{i}\omega\mu_0\left(\boldsymbol{J}_{\mathrm{ext}} + \delta\boldsymbol{J}_{\mathrm{ind}}^{(k)}\right), \tag{6}$$

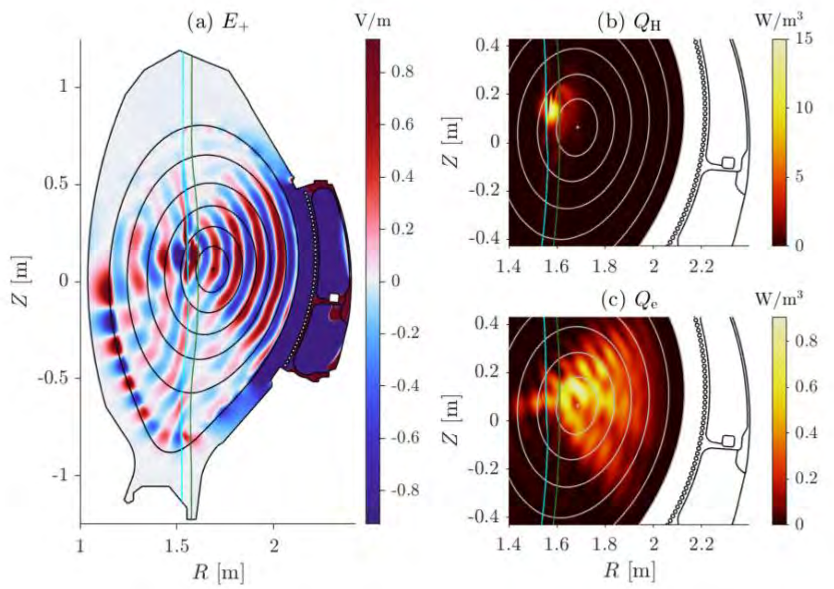

Figure 1. Left hand polarized electric field component (a), and power deposition on hydrogen ions (b) and electrons (c) as predicted by proposed iterative scheme.

where the operator A indicates Anderson acceleration, which is used to achieve convergence in the iterative scheme [7].

The iterative method described above has been implemented in the existing code FEMIC [8], which is a finite element model of radio-frequency wave propagation and dissipation fusion plasmas in the ion cyclotron range of frequencies (ICRF, ~50 MHz). Wave field and absorbed power densities are demonstrated for the fusion device ASDEX Upgrade in Figure 1. The method has been implemented in both 1D and 2D axisymmetric models and compared to existing ICRF codes with good results [6, 9], showing that it can model effects such as mode conversion to the electrostatic ion Bernstein wave, and to describe the dispersive narrowing (or broadening) of the resonance layer during minority ion cyclotron heating.

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